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ON THE CONFIGURATIONS OF CENTERS OF PLANAR HAMILTONIAN KOLMOGOROV CUBIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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We study the kind of centers that Hamiltonian Kolmogorov cubic polynomial differential systems can exhibit. Moreover, we analyze the possible configurations of these centers with respect to the invariant coordinate axes, and obtain that the real algebraic curve $xy(a+bx+cy+dx^2+exy+fy^2)=h$ has at most four families of level ovals in $\mathbb{R}^2$ for all real parameters a, b, c, d, e, f and h.

1. Introduction and statement of the main results

An equilibrium p of a differential system in $\mathbb{R}^2$ is a *center* if there is a neighborhood U of p such that p is the unique equilibrium in U and $U \setminus \{p\}$ is filled by periodic orbits surrounding p. The equilibrium p is a *focus* if there is a neighborhood U of p such that all the orbits in $U \setminus \{p\}$ spiral tending to p either in forward, or in backward time. These notions of center and focus go back to Poincaré [1881] and Dulac [1908].

In the qualitative theory of planar polynomial differential systems, the problem of distinguishing between a focus or a center (known simply as the *center-focus problem*), and the problem of knowing the possible configurations of centers are two very important topics, which are related to the Hilbert's 16th problem; see [Hilbert 1900; Ilyashenko 2002; Li 2003].

For the quadratic polynomial differential systems, the center-focus problem and the possible configurations of their centers were solved by Bautin [1952], Kapteyn [1911; 1912], Schlomiuk [1993], Vulpe [1983] and Żołądek [1994c]. However, these two problems are unsolved for cubic polynomial differential systems. There are many works on the centers for some different subclasses of cubic differential systems. For example, the centers of the cubic polynomial differential systems without quadratic terms have been determined by Malkin [1964], Vulpe and Sibirskii [1988],

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