A CLASSICAL FAMILY OF POLYNOMIAL ABEL DIFFERENTIAL EQUATIONS SATISFYING THE COMPOSITION CONJECTURE

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Abstract. We prove that the Composition Conjecture holds for the Abel differential equations, $y' = p(x)y^2+q(x)y^3$ where $p(x) \neq 0$ and $q(x)$ are complex polynomials satisfying $d(q(x)/p(x))/dx = cp(x)$ for some complex number c. This Abel differential equations appear in Kamke's book [13].

1. Introduction and statement of the result

We consider the differential system

$$\dot{x} = -y + f(x, y), \quad \dot{y} = x + g(x, y), \tag{1}$$

where f and g vanish at the origin together with their first derivatives. System (1) has a *center* at the origin if there is a neighborhood of it such that all the solutions (except the origin) are periodic. The classical *Center–Focus Problem* consists in determining necessary and sufficient conditions in order that system (1) has a center at the origin. In general this problem is unsolved, many deep partial results have been obtained (see [5]), but for instance are unknown the center conditions when f and g are polynomials of degree 3.

Here we consider the following variant of the Center–Focus Problem closely related to the original one. Let $p(x) \neq 0$ and $q(x)$ be complex polynomials. Then the Abel differential equation

$$y'(x) = \frac{dy}{dx} = p(x)\, y^2 + q(x)\, y^3 \tag{2}$$

has a *center* at a pair of complex numbers $\{a, b\}$ with $a \neq b$ if $y(a) = y(b)$ for any solution $y(x)$ with initial condition $y(a)$ sufficiently small.

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