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LIMIT CYCLES FOR A CLASS OF THREE-DIMENSIONAL POLYNOMIAL DIFFERENTIAL SYSTEMS

JAUME LLIBRE and JIANG YU

ABSTRACT. Perturbing the system $\dot{x} = -y(1+x)$, $\dot{y} = x(1+x)$, $\dot{z} = 0$ inside the family of polynomial differential systems of degree n in $\mathbb{R}^3$, we obtain at most n^2 limit cycles using the first-order averaging theory. Moreover, there exist such perturbed systems having at least n^2 limit cycles.

1. Introduction and statement of the main results

We perturb the system $\dot{x} = -y(1+x)$, $\dot{y} = x(1+x)$, $\dot{z} = 0$ inside a class of polynomial differential systems of degree n in $\mathbb{R}^3$. We note that the unperturbed system has the straight line $x = 0$, $y = 0$ and the plane $x = -1$ filled by singular points. Moreover, each plane $z = z_0 = \text{const}$ is invariant with respect to the flow of the unperturbed system. In fact, on every plane $z = z_0$, the singular point $(0, 0, z_0)$ is a center.

Theorem 1. *We consider the family of systems*

$$\begin{aligned} \dot{x} &= -y(1+x) + \varepsilon(ax + F(x,y,z)), \\ \dot{y} &= x(1+x) + \varepsilon(ay + G(x,y,z)), \\ \dot{z} &= \varepsilon(cz + R(x,y,z)), \end{aligned} \tag{1}$$

where $F(x,y,z)$, $G(x,y,z)$, and $R(x,y,z)$ are polynomials of degree n starting from terms of degree 2. Then there exists sufficiently small $\varepsilon_0 > 0$ such that, for $|\varepsilon| < \varepsilon_0$, there exist systems (1) having at least n^2 limit cycles bifurcating from the periodic orbits of the system $\dot{x} = -y(1+x)$, $\dot{y} = x(1+x)$, $\dot{z} = 0$.

Theorem 1 improves the results of [1], where, perturbing the system $\dot{x} = -y$, $\dot{y} = x$, $\dot{z} = 0$ inside the same class of polynomial vector fields, the averaging method up to the first order only can obtain at most $n(n-1)/2$

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