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On the Periodic Orbits of the Static, Spherically Symmetric Einstein-Yang-Mills Equations

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Abstract: In this paper we analyze the existence of the periodic orbits of the static, spherically symmetric Einstein–Yang–Mills Equations by using the qualitative theory of the ordinary differential equation. We prove that there are no periodic orbits restricted to some invariant set of codimension 1. Furthermore if there is a periodic orbit out of this invariant set, then there must be other periodic orbits, which are symmetric to the first one. We also have results on the non–existence of periodic orbits when the cosmological constant is negative.

1. Introduction

In this paper, we consider the static, spherically symmetric Einstein–Yang–Mills Equations (EYM) with a cosmological constant $a \in \mathbb{R}$,

$$\begin{aligned} \dot{r} &= rN, \\ \dot{W} &= rU, \\ \dot{N} &= (k-N)N - 2U^2, \\ \dot{k} &= s(1-2ar^2) + 2U^2 - k^2, \\ \dot{U} &= sWT + (N-k)U, \\ \dot{T} &= 2UW - NT, \end{aligned} \tag{1}$$

where $r, W, N, k, U, T \in \mathbb{R}^6$, $s \in \{-1; 1\}$ refers to regions, and the dot denotes a derivative with respect to the space–time variable t, see [2] for additional details on these equations. Let

$$f = 2kN - N^2 - 2U^2 - s(1 - T^2 - ar^2).$$

Then it holds that

$$\frac{df(t)}{dt} = -2N(t)f(t).$$