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GLOBAL PHASE PORTRAITS FOR QUADRATIC SYSTEMS WITH A HYPERBOLA AND A STRAIGHT LINE AS INVARIANT ALGEBRAIC CURVES

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ABSTRACT. In this article we consider a class of quadratic polynomial differential systems in the plane having a hyperbola and a straight line as invariant algebraic curves, and we classify all its phase portraits. Moreover these systems are integrable and we provide their first integrals.

1. INTRODUCTION AND STATEMENT OF MAIN RESULTS

In this article we consider the planar quadratic differential system

$$\begin{aligned} \dot{x} &= P(x,y), \\ \dot{y} &= Q(x,y), \end{aligned} \tag{1.1}$$

where P and Q are real polynomials such that the maximum of the degree of P and Q is 2. The dot in system (1.1) denotes derivative with respect to the independent variable t. We introduce some definitions.

Let f is a nonconstant polynomial in the variable x and y. The algebraic curve $f(x,y)=0$ is *an invariant curve* of system (1.1), if there exists some polynomial K(x,y) such that

$$\mathcal{X}(f) = P\frac{\partial f}{\partial x} + Q\frac{\partial f}{\partial y} = Kf,$$

and $K(x,y)$ is called the cofactor of the invariant curve $f(x,y)=0$.

Let $H(x,y)$ be a function defined in a dense and open subset U of $\mathbb{R}^2$. The function $H(x,y)$ is a *first integral* of system (1.1) if H is constant on the solutions of system (1.1) contained in U, i.e.

$$\mathcal{X}(H)\big|_U = P\frac{\partial H}{\partial x}(x,y) + Q\frac{\partial H}{\partial y}\big|_U = 0.$$

And a quadratic system is *integrable* in U if it has a first integral H in U.

Up to now several hundred of papers have been published studying differential aspects of quadratic systems, as their integrability, their limit cycles, their global dynamical behavior, and $\cdots$, see for instance the references quoted in the books of

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