LIMIT CYCLES COMING FROM THE PERTURBATION OF 2-DIMENSIONAL CENTERS OF VECTOR FIELDS IN $\mathbb{R}^3$

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ABSTRACT. In this paper we study the limit cycles of polynomial vector fields in $\mathbb{R}^3$ which bifurcates from three different kinds of two dimensional centers (nondegenerate and degenerate). The study is down using the averaging theory.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

One of the main problems in the theory of differential equations is the study of their periodic orbits, their existence, their number and their stability. As usual a limit cycle of a differential equation is a periodic orbit isolated in the set of all periodic orbits of the differential equation.

In this paper we shall study the limit cycles which bifurcate from the periodic orbits of three kinds of different 2–dimensional centers contained in a differential system of $\mathbb{R}^3$ when we perturb it. These kinds of bifurcations have been studied extensively for 2–dimensional systems (see for instance the book [5] and the references quoted there), but for 3–dimensional systems there are very few results, see for instance [1, 2, 6, 7].

First we shall study the perturbation of a 2–dimensional linear center, and after the perturbation of two degenerate 2–dimensional centers.

We consider the following system

$$\begin{aligned} \dot{x} &= -y + \varepsilon P(x,y,z), \\ \dot{y} &= \ \ x + \varepsilon Q(x,y,z) + \varepsilon \cos t, \\ \dot{z} &= \ az + \varepsilon R(x,y,z), \end{aligned} \tag{1}$$

where $a \neq 0$ and

$$P = \sum_{i+j+k=0}^{n} a_{i,j,k} x^i y^j z^k, \quad Q = \sum_{i+j+k=0}^{n} b_{i,j,k} x^i y^j z^k \quad R = \sum_{i+j+k=0}^{n} c_{i,j,k} x^i y^j z^k.$$

System (1) has been studied in [4] when $a = 0$ and without the perturbation due to $\varepsilon \cos t$. Then the unperturbed system has all $\mathbb{R}^3$ except the z–axis filled by periodic orbits. Here our unperturbed system (1) with $\varepsilon = 0$ only has the plane $z = 0$ except the origin filled of periodic orbits. The center on the plane $z = 0$ is called *nondegenerate* when the eigenvalues of its linear part are nonzero and

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