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LIMIT CYCLES FOR A CLASS OF THIRD-ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we study the limit cycles of the third-order differential equation $\dddot{x} - \mu\ddot{x} + \dot{x} - \mu x = \varepsilon F(x, \dot{x}, \ddot{x}, t)$, where $\mu \neq 0$, ε is small enough and $F \in \mathcal{C}^2$ is a 2π-periodic function of variable t.

1. Introduction and statement of the main results. One of the main problems in the theory of differential equations is the study of their periodic orbits, their existence, their number and their stability. As usual, a limit cycle of a differential equation is a periodic orbit isolated in the set of all periodic orbits of the differential equation.

In this paper we study the limit cycles of the following class of third-order ordinary differential equations

$$\dddot{x} - \mu\ddot{x} + \dot{x} - \mu x = \varepsilon F(x, \dot{x}, \ddot{x}, t), \tag{1}$$

where $\mu \neq 0$, and the dot means derivative with respect to the variable t, ε is small enough and $F \in \mathcal{C}^2$ is a 2π-periodic function of variable t. Here the variables x and t, and the parameters μ and ε are real.

There are many papers studying periodic orbits of third-order differential equations. Thus, our class of equations is not far from the ones studied in [**3, 13**]. But our main tool for studying the periodic orbits of equation (1) is completely different from the tools of the aforementioned papers. We shall use the averaging theory, more precisely Theorem 5 of the Appendix. Many of the papers dealing with periodic orbits of

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