On the Limit Cycles of the Polynomial Differential Systems with a Linear Node and Homogeneous Nonlinearities

Jaume Llibre

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain jllibre@mat.uab.cat

Jiang Yu

Department of Mathematics, Shanghai Jiao Tong University, Shanghai 200240, P. R. China jiangyu@sjtu.edu.cn

Xiang Zhang^{*} Department of Mathematics, and MOE-LSC, Shanghai Jiao Tong University, Shanghai 200240, P. R. China xzhang@sjtu.edu.cn

Received August 27, 2013; Revised December 27, 2013

We consider the class of polynomial differential equations $\dot{x} = \lambda x + P_n(x, y)$, $\dot{y} = \mu y + Q_n(x, y)$ in \mathbb{R}^2 where $P_n(x, y)$ and $Q_n(x, y)$ are homogeneous polynomials of degree n > 1 and $\lambda \neq \mu$, i.e. the class of polynomial differential systems with a linear node with different eigenvalues and homogeneous nonlinearities. For this class of polynomial differential equations, we study the existence and nonexistence of limit cycles surrounding the node localized at the origin of coordinates.

Keywords: Polynomial differential equations; node; limit cycle; homogeneous nonlinearities.

1. Introduction and Statement of the Main Results

A two-dimensional *polynomial differential system* in \mathbb{R}^2 is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \quad \frac{dy}{dt} = \dot{y} = Q(x, y), \quad (1)$$

where the dependent variables x and y, and the independent one (the time) t are real, and P(x, y)

and Q(x, y) are polynomials in the variables x and y with real coefficients. The *degree* of the polynomial system is $m = \max\{\deg P, \deg Q\}$. Recall that a *limit cycle* of a system (1) is an isolated periodic solution in the set of all periodic solutions of system (1).

One of the more difficult problems in the qualitative theory of polynomial differential equations in the plane \mathbb{R}^2 is the study of their limit cycles. Thus a classical problem related to these polynomial

^{*}Author for correspondence