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On polynomial integrability of the Euler equations on $\mathfrak{so}(4)$

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ABSTRACT

In this paper we prove that the Euler equations on the Lie algebra $\mathfrak{so}(4)$ with a diagonal quadratic Hamiltonian either satisfy the Manakov condition, or have at most four functionally independent polynomial first integrals.

1. Introduction and statement of the main results

Given a system of ordinary differential equations depending on parameters, it is in general very difficult to recognize for which values of the parameters the equations have first integrals. Except for some simple cases, this problem is very hard and there are no satisfactory methods to solve it.

In this paper we study the first integrals of the Euler differential equations on $\mathfrak{so}(4)$ in $\mathbb{R}^6$ depending on six parameters. Because these equations are used here exclusively as an interesting and nontrivial example of a multiparameter family of ordinary differential equations, we consider them without any explanations of their history, which can be found for instance in the references [1–6]. Here we discuss neither their physical origin, nor their relevant information that can be found in the quoted references.

Before introducing the Euler equations on $\mathfrak{so}(4)$ we first recall some basic definitions on the analytic and polynomial integrability of the polynomial differential systems of the form

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \mathbf{P}(\mathbf{x}), \quad \mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n, \tag{1}$$

with $\mathbf{P}(\mathbf{x}) = (P_1(\mathbf{x}), \ldots, P_n(\mathbf{x}))$ and $P_i \in \mathbb{R}[x_1, \ldots, x_n]$ for $i = 1, \ldots, n$. As usual $\mathbb{R}$ denotes the set of real numbers, and $\mathbb{R}[x_1, \ldots, x_n]$ denotes the polynomial ring over $\mathbb{R}$ in the variables $x_1, \ldots, x_n$.

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