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Bicentric quadrilateral central configurations

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ABSTRACT

A bicentric quadrilateral is a tangential cyclic quadrilateral. In a tangential quadrilateral, the four sides are tangents to an inscribed circle, and in a cyclic quadrilateral the four vertices lie on a circumscribed circle. In this paper, we classify all planar central configurations of the 4-body problem, where the four bodies are at the vertices of a bicentric quadrilateral.

1. Introduction and statement of the results

The well-known Newtonian n-body problem concerns with the motion of n mass points with positive mass m_i moving under their mutual attraction in $\mathbb{R}^d$ in accordance with Newton's law of gravitation.

The equations of the motion of the n-body problem are:

$$\ddot{x}_i = -\sum_{j=1, j\neq i}^{n} \frac{m_j(x_i - x_j)}{r_{ij}^3}, \quad 1 \leq i \leq n,$$

where we have taken the unit of time in such a way that the Newtonian gravitational constant be one, and $x_i \in \mathbb{R}^d (i = 1, \ldots, n)$ denotes the position vector of the i-body, $r_{ij} = |x_i - x_j|$ is the Euclidean distance between the i-body and the j-body.

Alternatively the equations of the motion can be written

$$m_i\ddot{x}_i = \nabla_i U(x), \quad 1 \leq i \leq n,$$

where $x = (x_1, \ldots, x_n)$, and

$$U(x) = \sum_{1\leq i<j\leq n} \frac{m_i m_j}{|x_i - x_j|},$$

is the potential of the system.

The solutions of the 2-body problem (also called the Kepler problem) has been completely solved. Unfortunately the solutions for the n-body ($n \geq 3$) is still an open problem.

For the Newtonian n-body problem the simplest possible motions are such that the configuration is constant up to rotations and scaling. Only some special configurations of particles are allowed in such motions called homographic solutions. Wintner [36] called them central configurations.

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