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Periodic Orbits of the Planar Anisotropic Manev Problem and of the Perturbed Hydrogen Atom Problem

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Abstract
In this paper we use the averaging theory for studying the periodic solutions of the planar anisotropic Manev problem and of two perturbations of the hydrogen atom problem. When a convenient parameter is sufficiently small we prove that for every value $e \in (0, 1)$ a unique elliptic periodic solution with eccentricity e of the Kepler problem can be continued to the mentioned three problems.

Keywords Periodic solutions · Averaging theory · Anisotropic Manev problem · Hydrogen atom problem

1 Introduction and Statement of the Results

After the works of Poincaré [32] and Schwarzschild [37] on the periodic solutions of the Hamiltonian systems, the study of the periodic solutions of these systems has been the research of many authors. These last 50 years many works on these periodic solutions has been made numerically thanks to the computers. The analytical study of these periodic orbits is in general more difficult.

In the present paper we study analytically, using the averaging theory, the periodic solutions of three Hamiltonian systems which are perturbation of the Kepler problem. The family of periodic orbits that we find in the three Hamiltonian systems here analyzed as far as we know are new, we cannot find in the literature any result on the periodic solutions here analyzed.

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