INVARIANT ALGEBRAIC SURFACES OF THE RIKITAKE SYSTEM

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Abstract

In this paper we use the method of characteristic curves for solving linear partial differential equations to study the invariant algebraic surfaces of the Rikitake system

$$\dot{x} = -\mu x + y(z+\beta), \quad \dot{y} = -\mu y + x(z-\beta), \quad \dot{z} = \alpha - xy.$$

Our main results are the following. First, we show that the cofactor of any invariant algebraic surface is of the form $rz + c$, where r is an integer. Second, we characterize all invariant algebraic surfaces. Moreover, as a corollary we characterize all values of the parameters for which the Rikitake system has a rational or algebraic first integral.

1. Introduction and statement of the main results

We consider the Rikitake systems

$$\begin{array}{rclcl} \dot{x} & = & -\mu x + y(z+\beta) & = & P(x,y,z), \\ \dot{y} & = & -\mu y + x(z-\beta) & = & Q(x,y,z), \\ \dot{z} & = & \alpha - xy & = & R(x,y,z), \end{array}$$

which is a simple model for describing the earth's magnetohydrodynamic dynamo (see for instance [2]), where x, y and z are real variables; α, β and μ are real parameters. These systems have been investigated as dynamical systems. For instance, Barge [1] gave conditions for which the system has two invariant surfaces. Hardy and Steeb [8] derived the conditions to find

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