INVARIANT ALGEBRAIC SURFACES OF THE BELOUSOV–ZHABOTINSKII SYSTEMS

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Abstract

In this paper we study the Field–Noyes model of the Belousov–Zhabotinskii chemical reaction:

$$\frac{dx}{d\tau} = s(y - xy + x - qx^2), \quad \frac{dy}{d\tau} = s^{-1}(-y - xy + rz), \quad \frac{dz}{d\tau} = w(x - z),$$

from the integrability point of view, and we obtain the necessary and sufficient conditions in order that this system has invariant algebraic surfaces, polynomial first integrals, rational first integrals, and invariants (also called integrals of motion).

1. Introduction and statement of the main results

The Belousov–Zhabotinskii chemical reaction, discovered in 1959 by Belousov [1], is one of the most interesting and best understood chemical oscillators. This reaction has been investigated intensively as a dynamic system (see for instance, [10], [13], [23], [25] and [26]). In 1974 Field and Noyes [8] abstracted a simpler model from the Belousov–Zhabotinskii chemical reaction which appears to retain the important features of the complete system. The differential equations describing the dynamic of the model are

$$\begin{aligned} \frac{dX}{dt} &= k_1AY - k_2XY + k_3BX - 2k_4X^2, \\ \frac{dY}{dt} &= -k_1AY - k_2XY + fk_5Z, \\ \frac{dZ}{dt} &= k_3BX - k_3Z, \end{aligned}$$

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