

ON THE NUMBER OF LIMIT CYCLES FOR SOME PERTURBED HAMILTONIAN POLYNOMIAL SYSTEMS

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Abstract

In this paper, we consider the perturbations of two Hamiltonian centers with Hamiltonians

$$H(x, y) = \frac{1}{2n}x^{2n} + \frac{1}{2m}y^{2m}, \quad H(x, y) = \frac{1}{2}y^2 + \frac{1}{2}x^2 + \frac{1}{2m}x^{2m},$$

respectively. For the former, we give the greatest number of isolated zeros (taking into account their multiplicity) of a class of Abelian integrals related to the corresponding perturbed Hamiltonian systems, and consequently obtain the indicated number of limit cycles from the perturbations of the corresponding Hamiltonian center in the class of differential polynomial systems. For the latter, we give the relative cohomology decomposition of the corresponding polynomial one form, and so obtain an estimate number of isolated zeros of the corresponding Abelian integral. We also study the maximum number of limit cycles that the perturbed systems can have surrounding a singular point.

1. Introduction

By definition a *polynomial system* is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = R(x, y), \quad \frac{dy}{dt} = \dot{y} = S(x, y), \quad (1)$$

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