Houston Journal of Mathematics

Volume 76, No. 1, 2050

TOPOLOGICAL PHASE PORTRAITS OF PLANAR SEMI–LINEAR QUADRATIC VECTOR FIELDS

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Communicated by James Damon

Abstract. In this paper we solve completely the topological classification of the phase portraits for a class of semi–linear quadratic vector fields, i.e. the vector fields of the form $\mathbf{X} = (ax + by, Ax + By + Cx^2 + Dxy + Ey^2)$. As a corollary of our results we answer the problem proposed by Ye Yanqian at the end of §2 of [30]. Moreover, we prove that quadratic systems of class (I) in the chinese classification of quadratic systems have exactly 50 different topological phase portraits, which corrects the result that such quadratic systems have only 47 different topological phase portraits (see Theorem 12.3 of Ye [30]).

1. Introduction and Statement of the Main Results

By definition a *quadratic vector field* is a map $\mathbf{X} = (P(x,\, y), Q(x,\, y)) : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$, where P and Q are quadratic polynomials of $\mathbf{R}[x,\, y]$. As usual $\mathbf{R}[x,\, y]$ denotes the ring of polynomials in the variables x and y with real coefficients. The differential system associated to the quadratic vector field $\mathbf{X}$ is

$$\dot{x} = P(x,\, y), \qquad \dot{y} = Q(x,\, y),$$

in what follows simply called a *quadratic system*.

Quadratic systems have been investigated intensively, and more than 1000 papers have been published about these systems (see for instance [24], [29] and [30]). But it is an open problem to determine the topological phase portraits of all quadratic vector fields.

1991 *Mathematics Subject Classification.* 34C05, 58F14.

Key words and phrases. Semi–linear quadratic vector fields, phase portraits, rotated vector fields, bifurcation.

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