International Journal of Bifurcation and Chaos, Vol. 12, No. 2 (2002) 421–428


# DARBOUX INTEGRABILITY FOR THE RÖSSLER SYSTEM

JAUME LLIBRE
*Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 – Bellaterra, Barcelona, Spain*
*jllibre@mat.uab.es*

XIANG ZHANG
*Department of Mathematics, Shanghai Jiaotong University, Shanghai 200030, P.R. China*
*m_x_zhang@263.net*

Received November 13, 2000; Revised May 24, 2001

In this note we characterize all generators of Darboux polynomials of the Rössler system by using weight homogeneous polynomials and the method of characteristic curves for solving linear partial differential equations. As a corollary we prove that the Rössler system is not algebraically integrable, and that every rational first integral is a rational function in the variable $x^2+y^2+2z$. Moreover, we characterize the topological phase portrait of the Darboux integrable Rössler system.

## 1. Introduction

We consider the Rössler system

$$\begin{aligned}\dot{x} &= -(y+z) &&= p(x,y,z),\\ \dot{y} &= x+ay &&= q(x,y,z),\\ \dot{z} &= b-cz+xz &&= r(x,y,z),\end{aligned} \tag{1}$$

where $a$, $b$ and $c$ are real constants, the dot denotes the derivative of the variables with respect to the time $t$. This model is a famous dynamic system, which was constructed by Rössler [1976]. Related with this system there are more than 300 papers published, in which they mainly investigated the existence and stability of limit cycles and the chaos phenomena. For instance, setting $a = b = 0.2$, if $c = 2.3$, system (1) has a period one limit cycle. If $c = 3.3$, system (1) has a period two limit cycle. If $c = 5.3$, system (1) has a period three limit cycle. If $c = 6.3$, system (1) has deterministic chaos behavior. Some recent papers about the Rössler system are [Hsieh *et al.*, 1999; Sinha *et al.*, 2000; Terëkhin & Panfilova, 1999]. In this note we investigate the Darboux and algebraic integrability of the Rössler system and we characterize the generators of all its Darboux polynomials.

The theory of algebraic integrability is a classical one. It received the main contributions from [Darboux, 1878] and [Poincaré, 1891, 1897]. The former gave a link between the first integral and the number of invariant algebraic curves (also called Darboux polynomials) for planar polynomial vector fields. The latter was mainly interested in the rational first integrals and noticed the difficulty of obtaining an algorithm for the existence of Darboux polynomials. In dimension 3 the theory of algebraic integrability has been applied to several systems: Rikitake systems and Lorenz systems [Llibre & Zhang, 2000, 2002], Lotka–Volterra systems [Cairó & Llibre, 2000; Labrunie, 1996; Moulin Ollagnier, 1997].

This paper is organized as follows. The basic definitions and the statement of the main results are given in Sec. 2, their proofs are presented in Sec. 3.