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DARBOUX INTEGRABILITY OF REAL POLYNOMIAL VECTOR FIELDS ON REGULAR ALGEBRAIC HYPERSURFACES

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In this paper we extend the Darboux theory of integrability in $\mathbb{R}^n$ to the regular algebraic hypersurfaces. Then we apply the extended theory first to the 3-dimensional generalized cylinders $(\mathbb{S}^1) \times \mathbb{R}^{3-r}$ of $\mathbb{R}^4$ for $r = 0, 1, 2, 3$; and after to the n-dimensional sphere $\mathbb{S}^n$ of $\mathbb{R}^{n+1}$.

1. Introduction.

The algebraic theory of integrability is a classical one. In 1878. Darboux [8] provided a link between algebraic geometry and the search of first integrals, and showed how to construct the first integral of polynomial vector fields in $\mathbb{R}^2$ or $\mathbb{C}^2$ having sufficient invariant algebraic curves. It also received contributions from Poincaré [17], who mainly was interested in the rational first integrals.

Good extensions of the Darboux theory of integrability to polynomial systems in $\mathbb{R}^n$ or $\mathbb{C}^n$ are due to Jouanolou [11] and Weil [20] (see also [12]). In [3], [4], [5], [6], [7] and [14] the authors developed the Darboux theory of integrability essentially in $\mathbb{R}^2$ or $\mathbb{C}^2$ considering not only the invariant algebraic curves but also the exponential factors, the independent singular points and the multiplicity of the invariant algebraic curves. Recently, in [9] and [13] there are extensions of the Darboux theory of integrability to 2-dimensional surfaces.

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