

Invariant algebraic surfaces of the Lorenz system

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In this paper we provide a complete classification of the Darboux invariants, of the irreducible Darboux polynomials, of the rational first integrals and of the algebraic integrability for the classical Lorenz system $\dot{x} = s(y-x)$, $\dot{y} = rx - y - xz$, $\dot{z} = -bz + xy$. In the proofs, we use the weight homogeneous polynomials and the method of characteristic curves for solving linear partial differential equations. © 2002 American Institute of Physics. [DOI: 10.1063/1.1435078]

I. INTRODUCTION AND DEFINITIONS

The Lorenz system:

$$\dot{x} = s(y-x) = P(x,y,z),$$

$$\dot{y} = rx - y - xz = Q(x,y,z), \tag{1}$$

$$\dot{z} = -bz + xy = R(x,y,z),$$

is a famous dynamical model (see, e.g., Lorenz, 1963), where x , y , and z are real variables; and s , r , and b are real parameters. This system has been thoroughly investigated as a dynamical system (see, e.g., Sparrow, 1982). From the point of view of integrability it was also intensively studied using different integrability theories (see, e.g., Cairó and Hua, 1993; Giacomini, Repetto, and Zandron, 1991; Goriely, 1996; Gupta, 1993; Kús, 1983; Schwarz, 1985; Segur, 1982; Steeb, 1982; and Strelcyn and Wojciechowski, 1988).

The problem involving the algebraic integrability and the Darboux polynomials is classical and difficult. It mainly received contributions from Darboux (1878), and Poincaré (1981, 1987). The former gave a link between algebraic geometry and the search of first integrals, and showed how to construct the first integrals of a planar polynomial vector field having sufficient number of invariant algebraic curves. The latter was mainly interested in rational first integrals and noticed the difficulty for obtaining an algorithm to compute Darboux polynomials. For three-dimensional systems, Labrunie (1996) and Moulin Ollagnier (1997) characterized all polynomial first integrals of the (a,b,c) Lotka–Volterra system. Moulin Ollagnier (1999) studied its homogeneous rational first integrals. Giacomini, Repetto, and Zandron (1991) investigated the integrals of motion for three-dimensional non-Hamiltonian dynamical systems. Llibre and Zhang (2000) characterized all the invariant algebraic surfaces, the polynomial first integrals, the rational first integrals, the invariants, and the algebraic integrability for the Rikitake system.

For the Lorenz system only six independent Darboux invariants had been found:

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