

Polynomial first integrals for quasi-homogeneous polynomial differential systems

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Abstract

In 1996, Furta (Furta S D 1996 *Z. Angew. Math. Phys.* **47** 112–31) and Goriely (Goriely A 1996 *J. Math. Phys.* **37** 1871–93) proved, independently, the existence of a link between the Kowalevskaya exponents of quasi-homogeneous polynomial differential systems and the degree of their quasi-homogeneous polynomial first integrals. Here, we provide a new link.

In the particular case that a Kowalevskaya matrix associated with a quadratic homogeneous polynomial differential system is diagonalizable, an improvement of the link found by Furta and Goriely has been obtained by Tsygvintsev (Tsygvintsev A 2001 *J. Phys. A: Math. Gen.* **34** 2185–93) in 2001, who additionally proved for these systems that an arbitrary homogeneous polynomial first integral of a given degree is a linear combination of a fixed set of polynomials. We show that Tsygvintsev's results are also true when this system has no diagonalizable Kowalevskaya matrices.

Finally, we characterize in terms of the Kowalevskaya exponents the two-dimensional quasi-homogeneous polynomial differential systems of weight degree 2 which have a quasi-homogeneous polynomial first integral.

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1. Introduction and statement of the main results

In this paper, we deal with polynomial differential systems of the form

$$\frac{dx}{dt} = \dot{x} = P(x), \quad x = (x_1, \dots, x_n) \in \mathbb{C}^n, \quad (1)$$