IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **40** (2007) 14207–14222 doi:10.1088/1751-8113/40/47/012

Algebraic limit cycles in polynomial systems of differential equations*

Jaume Llibre[1] **and Yulin Zhao**[2]

[1] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[2] Department of Mathematics, Sun Yat-sen University, Guangzhou 510275, People's Republic of China

E-mail: jllibre@mat.uab.cat and mcszyl@mail.sysu.edu.cn

Received 30 May 2007, in final form 15 October 2007
Published 6 November 2007
Online at stacks.iop.org/JPhysA/40/14207

Abstract
Using elementary tools we construct cubic polynomial systems of differential equations with algebraic limit cycles of degrees 4, 5 and 6. We also construct a cubic polynomial system of differential equations having an algebraic homoclinic loop of degree 3. Moreover, we show that there are polynomial systems of differential equations of arbitrary degree that have algebraic limit cycles of degree 3, as well as give an example of a cubic polynomial system of differential equations with two algebraic limit cycles of degree 4.

PACS number: 02.30.Hq
Mathematics Subject Classification: 34C05, 34A34, 34C14

1. Introduction

By definition, a *planar polynomial system of differential equations* is a system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \qquad \frac{dy}{dt} = \dot{y} = Q(x, y), \tag{1}$$

where P and Q are real polynomials in the variables x and y. The *degree* i of the polynomial system of differential equations is the maximum of the degrees of the polynomials P and Q. In what follows, a planar polynomial system of differential equations of degree 2 or 3 will be called simply a *quadratic* or a *cubic* system, respectively.

In 1900, Hilbert [16] in the second part of its 16th problem proposed to find an estimation of the uniform upper bound for the number of limit cycles of all polynomial systems of differential equations of a given degree, and also to study their distribution or configuration

* The first author is partially supported by a DGICYT/FEDER grant number MTM2005-06098-C02-01 and by a CICYT grant number 2005SGR 00550. The second author is partially supported by the Spanish grant SAB2005-0029, NSF of China (no 10571184) and SRF for ROCS, SEM.

