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On the algebraic limit cycles of Liénard systems

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Abstract

For the Liénard systems $\dot{x} = y$, $\dot{y} = -f_m(x)y - g_n(x)$ with f_m and g_n polynomials of degree m and n, respectively, we present explicit systems having algebraic limit cycles in the cases $m \geqslant 2$ and $n \geqslant 2m+1$ and $m \geqslant 3$ and $n = 2m$. Also we prove that the Liénard system for $m = 3$ and $n = 5$ has no hyperelliptic limit cycles. This shows that the result of theorem 1(c) of Zoladek (1998 *Trans. Am. Math. Soc.* **350** 1681–701) on the existence of algebraic limit cycles of the Liénard system is not correct. Moreover, we characterize all hyperelliptic limit cycles of the Liénard systems for $m = 4$ and $n = 6$ or $n = 7$. For $m > 4$ and $n = 2m-1$ or $n = 2m-2$ we prove that there are Liénard systems which have $[m/2]-1$ algebraic limit cycles, where the $[\cdot]$ denotes the integer part function.

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1. Introduction and statement of the main results

One of the more difficult problems in the qualitative theory of planar differential systems is the study of their *limit cycles*, i.e. the study of isolated periodic orbits inside the set of all periodic orbits, see for instance Hilbert's 16th problem [4]. Since this problem looks, at the moment, intractable, here we simplify it in two directions. First, instead of working with all polynomial differential systems in $\mathbb{R}^2$ we restrict our attention to the subclass of polynomial Liénard systems and, second, instead of dealing with limit cycles we only consider the *algebraic limit cycles*, i.e. the limit cycles contained in algebraic curves, see for instance [2].

Consider the Liénard system

$$\dot{x} = y, \qquad \dot{y} = -f_m(x)y - g_n(x), \tag{1}$$