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Darboux theory of integrability for polynomial vector fields in \mathbb{R}^n taking into account the multiplicity at infinity

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Abstract

Darboux theory of integrability was established by Darboux in 1878, which provided a relation between the existence of first integrals and invariant algebraic hypersurfaces of vector fields in \mathbb{R}^n or \mathbb{C}^n with $n \ge 2$. Jouanolou 1979 improved this theory to obtain rational first integrals via invariant algebraic surfaces using sophisticated tools of algebraic geometry. Recently in [J. Llibre, X. Zhang, Darboux theory of integrability in \mathbb{C}^n taking into account the multiplicity, J. Differential Equations, in press] this theory was improved taking into account not only the invariant algebraic hypersurfaces but also their multiplicity. In this paper we will show that if the hyperplane at infinity for a polynomial vector field in \mathbb{R}^n has multiplicity larger than 1, we can improve again the Darboux theory of integrability. We also show some difficulties for obtaining an extension of this result to polynomial vector fields in \mathbb{C}^n .

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