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Rational first integrals in the Darboux theory of integrability in $\mathbb{C}^n$

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Abstract

In 1979 Jouanolou showed that if the number of invariant algebraic hypersurfaces of a polynomial vector field in $\mathbb{R}^n$ or $\mathbb{C}^n$ of degree d is at least $\binom{d+n-1}{n} + n$, then the vector field has a rational first integral. His proof used sophisticated tools of algebraic geometry. We provide an easy and elementary proof of Jouanolou's result using linear algebra.

MSC: 34A34; 34C05; 34C14

Keywords: Darboux integrability; Rational first integral

1. Introduction

Nonlinear ordinary differential equations appear in many branches of applied mathematics, physics and, in general, in applied sciences. For a differential system or a vector field defined in $\mathbb{R}^n$ or $\mathbb{C}^n$ the existence of a first integral reduces the study of its dynamics in one dimension; of course working with real or complex time, respectively. So a natural question is: *Given a vector field on $\mathbb{R}^n$ or $\mathbb{C}^n$, how to recognize if this vector field has a first integral*? This question has no a satisfactory answer up to now. Many different methods have been used for studying the existence of first integrals of vector fields. Some of these methods based on: Noether symmetries [4], the Darboux theory of integrability [7], the Lie symmetries [13], the Painlevé analysis [2], the use of

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