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On the Hopf-zero bifurcation of the Michelson system

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ABSTRACT

Applying a new result for studying the periodic orbits of a differential system via the averaging theory, we provide the first analytic proof of the existence of a Hopf-zero bifurcation for the Michelson system

$$\dot{x} = y, \qquad \dot{y} = z, \qquad \dot{z} = c^2 - y - \frac{x^2}{2},$$

at $c = 0$. Moreover our method estimates the shape of this periodic orbit as a function of $c > 0$, sufficiently small.

1. Introduction and statement of results

The Michelson system

$$\dot{x} = y, \qquad \dot{y} = z, \qquad \dot{z} = c^2 - y - \frac{x^2}{2}, \tag{1}$$

with $(x, y, z) \in \mathbb{R}^3$ and the parameter $c \geq 0$, was obtained by Michelson [1] in the study of the travelling wave solutions of the Kuramoto–Sivashinsky equation. It is well known that system (1) is reversible with respect to the involution $R(x, y, z) = (-x, y, -z)$ and is volume-preserving under the flow of the system. It is easy to check that system (1) has two finite singularities $S_1 = (-\sqrt{2}c, 0, 0)$ and $S_2 = (\sqrt{2}c, 0, 0)$ for $c > 0$, which are both saddle foci. The former has a two-dimensional stable manifold and the latter has a two-dimensional unstable manifold.

In what follows we give a brief summary of the dynamics results for system (1). The first study on system (1) goes back to Michelson [1] who proved that if $c > 0$ is sufficiently large, system (1) has a unique bounded solution, which is the transversal heteroclinic orbit connecting the two finite singularities. When c decreases there will appear a cocoon bifurcation, which was verified in [1–3] using computer assistance. As regards the appearance of the cocoon bifurcation, Remark 1.6 of [4] explained that if there exists a saddle-node periodic orbit γ at some value $c_0 > 0$ of the parameter which is symmetric with respect to the involution R, then for c on one side of c_0 the saddle-node periodic orbits split into two limit cycles, while for c on the other side of c_0 no periodic orbits will be present near γ, but a cocooning cascade will appear. Lamb et al. [5] proved that the Michelson system can have countably infinitely many heteroclinic loops, countably infinitely many homoclinic loops, and a countable infinity of hyperbolic base sets (horseshoes).

For $c > 0$ small, numerical experiments (see for instance [6]) and asymptotic expansions in sine series (see [1] in 1986 and [7] in 2003) revealed the existence of a Hopf-zero bifurcation at the origin for $c = 0$. But their results do not provide an analytic proof of the existence of such a Hopf-zero bifurcation. By a *Hopf-zero bifurcation* we mean that when $c = 0$ the Michelson system has the origin as a singularity having eigenvalues $0, \pm i$, and when $c > 0$ is sufficiently small the

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