On the Darboux Integrability of Polynomial Differential Systems

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Abstract This is a survey on recent results on the Darboux integrability of polynomial vector fields in \mathbb{R}^n or \mathbb{C}^n with $n \ge 2$. We also provide an open question and some applications based on the existence of such first integrals.

Keywords Polynomial differential systems · Polynomial vector fields · Darboux integrability

Mathematics Subject Classification (2000) 34A34 · 34C05 · 34C14

1 Introduction

In many branches of applied mathematics, physics and, in general, in applied sciences appear nonlinear ordinary differential equations. If a differential equation or vector field defined on a real or complex manifold has a first integral, then its study can be reduced by one dimension. Therefore a natural question is: *Given a vector field on a manifold, how to recognize if this vector field has a first integral defined on an open and dense subset of the manifold*? In general this question has no a good answer up to now.

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