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On the Limit Cycles of Linear Differential Systems with Homogeneous Nonlinearities

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Abstract. We consider the class of polynomial differential systems of the form $\dot{x} = \lambda x - y + P_n(x, y)$, $\dot{y} = x + \lambda y + Q_n(x, y)$, where P_n and Q_n are homogeneous polynomials of degree n. For this class of differential systems we summarize the known results for the existence of limit cycles, and we provide new results for their nonexistence and existence.

1 Introduction and Statement of the Main Results

One of the main problems in the qualitative theory of real planar differential systems is how to control the existence, non-existence, or uniqueness of limit cycles for a given class of polynomial differential systems.

Limit cycles of planar differential systems were defined by Poincaré [13] and started to be studied intensively at the end of the 1920s by van der Pol [14], Liénard [7], and Andronov [1].

In this work we study the real planar polynomial differential systems of the form

$$\dot{x} = \lambda x - y + P_n(x, y), \quad \dot{y} = x + \lambda y + +Q_n(x, y), \tag{1.1}$$

where P_n and Q_n are homogeneous polynomials of degree n.

In order to be more precise, we need to introduce some notation and basic results. Then, in polar coordinates (r, θ) defined by $x = r\cos\theta$, $y = r\sin\theta$, system (1.1) can be written as

$$\dot{r} = \lambda r + f(\theta) r^n, \quad \dot{\theta} = 1 + g(\theta) r^{n-1}, \tag{1.2}$$

where

$$f(\theta) = \cos\theta P_n(\cos\theta, \sin\theta) + \sin\theta Q_n(\cos\theta, \sin\theta),$$
$$g(\theta) = \cos\theta Q_n(\cos\theta, \sin\theta) - \sin\theta P_n(\cos\theta, \sin\theta)$$

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