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Limit cycles of linear vector fields on manifolds

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Abstract

It is well known that linear vector fields on the manifold $\mathbb{R}^n$ cannot have limit cycles, but this is not the case for linear vector fields on other manifolds. We study the periodic orbits of linear vector fields on different manifolds, and motivate and present an open problem on the number of limit cycles of linear vector fields on a class of $\mathcal{C}^1$ connected manifold.

Keywords: limit cycle, periodic orbit, centre, isochronous centre, averaging method
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1. Introduction and statement of the results

In qualitative theory, the periodic orbits play an important role in the study of the dynamics of ordinary differential equations or vector fields. Inside the periodic orbits there is a class of limit cycles; a *limit cycle* is a periodic orbit isolated in the set of all the periodic orbits of the differential equation or vector field.

Many pieces of work have been done on the limit cycles of many different differential equations (see for instance [6, 8, 9, 11, 15, 20] and the references quoted therein), but as far as we know nobody has paid any attention to the limit cycles of linear vector fields—probably because the vector fields on $\mathbb{R}^n$ have no limit cycles. But there are some interesting questions regarding the linear vector fields on other manifolds. Demonstrating some of these questions is the objective of this paper.

Linear vector fields are the easiest of the vector fields, but they play an important role in the theory of differential systems and also in applications, as the following four examples show.

