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Dynamics of Some Three-Dimensional Lotka–Volterra Systems

Jaume Llibre and Xiang Zhang

Abstract. We characterize the dynamics of the following two Lotka–Volterra differential systems:

$$\begin{aligned} \dot{x} &= x(r + ay + bz), \\ \dot{y} &= y(r - ax + cz), \\ \dot{z} &= z(r - bx - cy), \end{aligned} \quad \text{and} \quad \begin{aligned} \dot{x} &= x(r + ax + by + cz), \\ \dot{y} &= y(r + ax + dy + ez), \\ \dot{z} &= z(r + ax + dy + fz). \end{aligned}$$

We analyze the biological meaning of the dynamics of these Lotka–Volterra systems

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Keywords. Lotka–Volterra system, invariant, global dynamics, phase portrait.

1. Introduction and Statement of the Main Results

The Lotka–Volterra differential systems appeared at the work of A. J. Lotka in 1910 for modeling autocatalytic chemical reactions, and were extended by himself in 1920 to

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x(\alpha - \beta y), \quad \frac{\mathrm{d}y}{\mathrm{d}t} = y(-\gamma + \delta x),$$

for modeling the dynamics of a plant species and a herbivorous animal species, where x and y are, respectively, the numbers of preys and predators, and α, β, γ, and δ are positive real parameters describing the interaction of the two species. Volterra developed this last model independently from Lotka to explain the exchange of the fish catches between fish and predatory fish in the Adriatic Sea during the first World War. Kolmogorov [9] in 1936 studied these systems again and extended them to arbitrary dimension, and for this reason, these kinds of systems are also called *Kolmogorov systems*.

This last Lotka–Volterra differential system has been modified in different ways for studying the dynamics of the interaction between the competition of two or more species. Lotka–Volterra systems have also been used to model dynamical phenomena from different subjects, such as hydrodynamics [3], plasma physics [10], chemical reactions [7], and evolution of conflicting

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