

Limit cycles of the classical Liénard differential systems: A survey on the Lins Neto, de Melo and Pugh's conjecture

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Abstract

In 1977 Lins Neto et al. (1977) conjectured that the classical Liénard system

$$\dot{x} = y - F(x), \quad \dot{y} = -x,$$

with $F(x)$ a real polynomial of degree n , has at most $[(n - 1)/2]$ limit cycles, where $[\cdot]$ denotes the integer part function. In this paper we summarize what is known and what is still open on this conjecture. For the known results on this conjecture we present a complete proof.

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1. Introduction and statement of the main results

The classical Liénard system

$$\dot{x} = y - F(x), \quad \dot{y} = -x, \tag{1}$$

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