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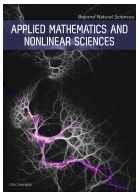

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On the integrability of the Hamiltonian systems with homogeneous polynomial potentials

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Abstract

We summarize the known results on the integrability of the complex Hamiltonian systems with two degrees of freedom defined by the Hamiltonian functions of the form

$$H = \frac{1}{2}\sum_{i=1}^{2} p_i^2 + V(q_1, q_2),$$

where $V(q_1, q_2)$ are homogeneous polynomial potentials of degree k.

Keywords: Hamiltonian system with 2–degrees of freedom, homogeneous polynomial potential, integrability.
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1 Introduction

In the theory of ordinary differential equations and in particular in the theory of Hamiltonian systems the existence of first integrals is important, because they allow to lower the dimension where the Hamiltonian system is defined. Furthermore, if we know a sufficient number of first integrals, these allow to solve the Hamiltonian system explicitly, and we say that the system is integrable. Almost until the end of the 19th century the major part of mathematicians and physicians believe that the equations of classical mechanics were integrable, and that to find their first integrals was mainly a computational problem. Now we know that the integrability is a rare phenomenon, and that in general it is not easy to know when a given Hamiltonian system is or not integrable.

The objective of this paper is to summarize the results that are known on the integrability of the complex

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