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Limit cycles created by piecewise linear centers

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ABSTRACT

In the last few years, the interest for studying the piecewise linear differential systems has increased strongly, mainly due to their applications to many physical phenomena. In the study of these differential systems, the limit cycles play a main role. Up to now, the major part of papers which study the limit cycles of the piecewise linear differential systems consider only two pieces. Here, we consider piecewise linear differential systems with three pieces. In this paper, we study the limit cycles of the discontinuous piecewise linear differential systems in the plane $\mathbb{R}^2$ formed by three arbitrary linear centers separated by the set $\Sigma = \{(x, y) \in \mathbb{R}^2 : y = 0 \text{ or } x = 0 \text{ and } y \geq 0\}$. We prove that such discontinuous piecewise linear differential systems can have 1, 2, or 3 limit cycles, with 3 the maximum number of limit cycles that such systems can have. Moreover, the limit cycles are nested and must intersect Σ in three or four points. The limit cycles having three intersection points with Σ can reach the maximum number 3. The limit cycles having four intersection points with Σ are at most 1, and if it exists, the systems could simultaneously have 1 or 2 limit cycles intersecting Σ in three points.

The piecewise linear differential systems appear in a natural way in physics, biology, and so on, where periodic phenomena play a relevant role. This paper studies the periodic motions of a class of piecewise linear differential centers and obtains the maximum number of isolated periodic orbits (i.e., limit cycles) for such a class of piecewise linear differential centers separated in three pieces.

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

A periodic orbit of a differential system in the plane $\mathbb{R}^2$, which is isolated in the set of all periodic orbits of the system, is called a "limit cycle." The limit cycles started to be studied at the end of the 19th century by Poincaré.[16] Later on, it was observed that the limit cycles modelize many phenomena of the real world, see, for instance, the Belousov–Zhabotinskii reaction,[2,22] the van der Pol oscillator,[17,18] or the motion of the galaxies,[5] and many examples can be found in the survey[14] or in the book.[3]

In the book of Andronov *et al.*,[1] there appeared some of the first studies on the "discontinuous" piecewise linear differential systems in the plane $\mathbb{R}^2$ separated by straight lines. The work on such differential systems has continued up today, mainly due to their applications, for instance, in mechanics, economy, electrical circuits, etc.; see the surveys[14,21] and the books.[3,20]

There are two types of limit cycles in the planar discontinuous piecewise linear differential systems, the crossing and sliding ones. The "sliding limit cycles" contain some arc of the lines of discontinuity that separate the different linear differential systems (more precise definition can be found in Ref. 15). The "crossing limit cycles" only contain isolated points of the lines of discontinuity. In this paper, we only consider the crossing limit cycles of some planar discontinuous piecewise linear differential systems separated by pieces of straight lines. From now on, we shall work with only crossing limit cycles, but we simply call them limit cycles instead of crossing limit cycles.

The easiest discontinuous piecewise linear differential systems in the plane are the discontinuous piecewise linear differential systems separated by a unique straight line. It is known that such differential systems can have 3 limit cycles; see Refs. 4, 6–9, and 11. However, at this moment, it is an open problem to know if 3 is the maximum number of limit cycles that such discontinuous piecewise linear differential systems can have. Also, limit cycles of piecewise linear differential systems in $\mathbb{R}^3$ have been analyzed; see, for instance, Ref. 12.

Here, our objective is to study the number of limit cycles, which can exhibit the planar discontinuous piecewise linear differential