Limit Cycles for Discontinuous Planar Piecewise Linear Differential Systems Separated by an Algebraic Curve

Jaume Llibre

Departament de Matemàtiques, Universitat Autònoma de Barcelona, Spain jllibre@mat.uab.cat

Xiang Zhang

School of Mathematical Sciences, Key Laboratory of Scientific and Engineering Computing (Ministry of Education), Shanghai Jiao Tong University, Shanghai 200240, P. R. China xzhang@sjtu.edu.cn

Received May 15, 2018

We study how to change the maximum number of limit cycles of the discontinuous piecewise linear differential systems with only two pieces in function of the degree of the discontinuity of the algebraic curve between the two linear differential systems. These discontinuous differential systems appear frequently in applied sciences.

Keywords: Limit cycle; discontinuous piecewise linear differential systems; discontinuity with an algebraic curve.

1. Introduction and Statement of the Main Result

The dynamics of the piecewise linear differential systems began to be studied around 1930, mainly in the book by Andronov *et al.* [1966]. Many researchers from different fields considered this kind of differential systems, because they are widely used to model phenomena appearing in mechanics, electronics, economy, neuroscience,..., see for more details and applications, the books by di Bernardo *et al.* [2008] and Simpson [2010], the surveys of Makarenkov and Lamb [2012], and Teixeira [2009], the articles [Izhikevich, 2007; Kokotović *et al.*, 1999; Léger & Pratt, 2011; Pratt *et al.*, 2010; Wechselberger & Weckesser, 2009], and the references which appear in all these works.

A periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system is a *limit cycle*. One of the main problems in the dynamics of the differential systems on the plane is to control the existence and the number of their limit cycles. This problem restricted to polynomial differential systems is the famous 16th Hilbert's problem, see more details in [Hilbert, 1900; Ilyashenko, 2002; Li, 2003].

These last years many authors have studied the limit cycles of discontinuous piecewise linear differential systems in \mathbb{R}^2 . Of course, the simplest piecewise linear differential systems in \mathbb{R}^2 are the ones having only two pieces separated by a curve, and when this curve is a straight line. Thus the limit cycles of this last class of discontinuous piecewise linear differential systems has been intensively studied, see [Artés *et al.*, 2013; Braga & Mello, 2013; Buzzi *et al.*, 2013; Euzébio & Llibre, 2015; Freire *et al.*, 1998; Freire *et al.*, 2012, 2014;