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The non-existence, existence and uniqueness of limit cycles for quadratic polynomial differential systems

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We provide sufficient conditions for the non-existence, existence and uniqueness of limit cycles surrounding a focus of a quadratic polynomial differential system in the plane.

Keywords: quadratic systems; quadratic polynomial differential system; limit cycles

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1. Introduction and statement of the main results

One of the main problems in the qualitative theory of real planar differential systems is controlling the existence, non-existence and uniqueness of limit cycles for a given class of polynomial differential systems.

Limit cycles of planar differential systems were defined by Poincaré [15–18], and started to be studied intensively at the end of the 1920s by van der Pol [22], Liénard [11] and Andronow [1].

It is well known that if a quadratic polynomial differential system, or simply a *quadratic system*, has one limit cycle, this must surround a focus of the system (see, for example, [7, proposition 8.13]), and according to Bautin [2] such a system can be written in the form

$$\left.\begin{aligned}\dot{x} &= \lambda_1 x - y - \lambda_3 x^2 + (2\lambda_2+\lambda_5)xy + \lambda_6 y^2,\\ \dot{y} &= x + \lambda_1 y + \lambda_2 x^2 + (2\lambda_3+\lambda_4)xy - \lambda_2 y^2.\end{aligned}\right\} \tag{1.1}$$

In order to state our results we write the quadratic system (1.1) in polar coordinates (r,θ), defined by $x = r\cos\theta$, $y = r\sin\theta$, and we get

$$\dot{r} = \lambda_1 r + f r^2, \qquad \dot{\theta} = 1 + g r, \tag{1.2}$$