DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS SERIES B Volume 24, Number 4, April 2019

pp. 1769–1784

LIMIT CYCLES OF DISCONTINUOUS PIECEWISE QUADRATIC AND CUBIC POLYNOMIAL PERTURBATIONS OF A LINEAR CENTER

JAUME LLIBRE

Departament de Matemàtiques Universitat Autònoma de Barcelona 08193 Bellaterra, Barcelona, Catalonia, Spain

YILEI TANG^{*}

School of Mathematical Sciences Shanghai Jiao Tong University Shanghai, 200240, China

(Communicated by Yuan Lou)

ABSTRACT. We apply the averaging theory of high order for computing the limit cycles of discontinuous piecewise quadratic and cubic polynomial perturbations of a linear center. These discontinuous piecewise differential systems are formed by two either quadratic, or cubic polynomial differential systems separated by a straight line.

We compute the maximum number of limit cycles of these discontinuous piecewise polynomial perturbations of the linear center, which can be obtained by using the averaging theory of order n for n = 1, 2, 3, 4, 5. Of course these limit cycles bifurcate from the periodic orbits of the linear center. As it was expected, using the averaging theory of the same order, the results show that the discontinuous quadratic and cubic polynomial perturbations of the linear center have more limit cycles than the ones found for continuous and discontinuous linear perturbations.

Moreover we provide sufficient and necessary conditions for the existence of a center or a focus at infinity if the discontinuous piecewise perturbations of the linear center are general quadratic polynomials or cubic quasi-homogenous polynomials.

1. Introduction and statement of the main results. The interest on the dynamics of piecewise linear differential systems essentially started with the book of Andronov et al [1], whose Russian version appeared around the 1930's. Due to the rich dynamics of the piecewise linear differential systems, and their applications in mechanics, electronics, economy, neuroscience, ..., these systems have been studied by researchers from many different fields, see for instance the books of Bernardo et al [4] and of Simpson [31], the survey of Makarenkov and Lamb [28], and the references mentioned in all these works.

For the planar *continuous* piecewise linear differential systems with two zones separated by a straight line, Lum and Chua [26, 27] in 1991 conjectured that such

²⁰¹⁰ Mathematics Subject Classification. 34C29, 34C25, 34C05.

Key words and phrases. Periodic solution, limit cycle, discontinuous piecewise differential system, averaging theory.