DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Volume **25**, Number **4**, December **2009**

doi:10.3934/dcds.2009.25.1081

pp. **1081–1108**

DETECTING ALIEN LIMIT CYCLES NEAR A HAMILTONIAN 2-SADDLE CYCLE

Stijn Luca and Freddy Dumortier

Universiteit Hasselt, Campus Diepenbeek
Agoralaan - Gebouw D
B-3590 Diepenbeek, Belgium

Magdalena Caubergh

Departament de Matemàtiques, Edifici C,
Universitat Autònoma de Barcelona, 08193 Cerdanyola de Vallès
Barcelona, Spain

Robert Roussarie

Institut de Mathématique de Bourgogne, U.M.R. 5584 du C.N.R.S.
Université de Bourgogne, B.P. 47 870
21078 Dijon Cedex, France

Abstract. This paper aims at providing an example of a cubic Hamiltonian 2-saddle cycle that after bifurcation can give rise to an alien limit cycle; this is a limit cycle that is not controlled by a zero of the related Abelian integral. To guarantee the existence of an alien limit cycle one can verify generic conditions on the Abelian integral and on the transition map associated to the connections of the 2-saddle cycle. In this paper, a general method is developed to compute the first and second derivative of the transition map along a connection between two saddles. Next, a concrete generic Hamiltonian 2-saddle cycle is analyzed using these formula's to verify the generic relation between the second order derivative of both transition maps, and a calculation of the Abelian integral.

1. **Introduction and settings.** We deal with perturbations of Hamiltonian systems:

$$(X_{(\overline{\mu},\varepsilon)}):\begin{cases}\dot{x} &= -\frac{\partial H}{\partial y}+\varepsilon f,\\ \dot{y} &= \frac{\partial H}{\partial x}+\varepsilon g,\end{cases}\tag{1}$$

where $H(x,y), f(x,y,\overline{\mu},\varepsilon), g(x,y,\overline{\mu},\varepsilon)$ are C^∞ functions, ε is considered to take small positive values and $\overline{\mu}$ varies in some compact subset $K\subset I\!R^p$. Further we abbreviate $\mu=(\overline{\mu},\varepsilon)$.

We suppose that the flow of $X_{(\overline{\mu},0)}=X_H$ contains *a period annulus* bounded by a hyperbolic 2–saddle cycle $\mathcal{L}$ as in Figure 1. A period annulus is a subset of the plane filled by closed orbits of X_H. The hyperbolic 2-saddle cycle consists of two saddle-connections Γ_1 and Γ_2 and two hyperbolic saddles s_1 and s_2 such that $s_1:=\alpha(\Gamma_1)=\omega(\Gamma_2)$ and $s_2:=\alpha(\Gamma_2)=\omega(\Gamma_1)$. We choose H to be zero on the 2-saddle cycle and strictly positive on the nearby closed orbits.

In [9] it is proven that, for $\overline{\mu}\in K$ and $\varepsilon>0$ near zero, $\mathcal{L}$ can produce limit cycles that are not controlled by zeros of the related Abelian integral (cfr. (5));

2000 *Mathematics Subject Classification.* 34C23, 34C25, 34E10, 34C25, 34C20, 34C08.

Key words and phrases. Planar vector field, Hamiltonian perturbation, limit cycle, Abelian integral, two-saddle cycle, alien limit cycle, transition map.