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On the Existence of Polynomial Inverse Integrating Factors in Quadratic Systems with Limit Cycles

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Abstract. In this paper we consider planar quadratic polynomial vector fields that can have limit cycles. We study the nonexistence of invariant algebraic curves, polynomial inverse integrating factors and algebraic limit cycles of arbitrary degree for these systems. We conclude that there are not algebraic limit cycles except for $\ell N\delta \neq 0$ and $M^2 - 4\ell N \geq 0$ in family (I) of Ye Yian Qian, i.e., $\dot{x} = \delta x - y + \ell x^2 + Mxy + Ny^2$, $\dot{y} = x$. We also prove that the polynomial inverse integrating factors into families (I), $(II)_{N=0}$, $(III)_{a=0}$ and $(III)_{N=0}$ generically have at most degree 3. So, in the studied cases, the existence of polynomial inverse integrating factor implies the nonexistence of limit cycles or at most the existence of a circle as a unique limit cycle.
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1 Introduction

Let us consider a *planar polynomial differential system* of the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \dot{x} = P(x,y) = \sum_{k=0}^{m} P_k(x,y) \ , \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \dot{y} = Q(x,y) = \sum_{k=0}^{m} Q_k(x,y) \ , \quad (1)$$

in which P, $Q \in \mathbb{R}[x,y]$ are relative prime polynomials in the variables x and y and P_k and Q_k are homogeneous polynomials of degree k. Throughout this paper we will denote by $m = \max\{\deg P, \deg Q\}$ the *degree* of system (1).

One interesting question to ask is whether some invariant curve of system (1) is algebraic, i.e. can be described implicitly by $f(x,y) = 0$ where f is a polynomial. In general, the answer is not easy but it is very interesting because it is known that the existence of *invariant algebraic curves* can be used to prove the existence or nonexistence of limit cycles of system (1). In short, invariant algebraic curves and integrability have a narrow relationship for planar polynomial systems like it is clearly shown in the Darboux theory