## Conservation Laws in Biochemical Reaction Networks

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## Abstract

We study the existence of linear and non-linear conservation laws in biochemical reaction networks with mass-action kinetics. It is straightforward to identify the linear conservation laws as they are related to the left null-space of the stoichiometry matrix. The non-linear conservation laws are much more difficult to study and so far have rarely been considered in the context of mass-action reaction networks. Our aim is to give structural conditions – that is, parameter independent conditions, on a reaction network to guarantee the existence of nonlinear conservation laws of some type. We do so by means of Darboux theory of integrability. We show that  $F(x) = x_i$  is a Darboux polynomial if and only if the reaction network fulfil a certain structural condition. Furthermore, this allows us to conclude that a specific type of a non-linear first integral (similar to that of the Lotka-Volterra system) only exists if the reaction network fulfils the same structural condition. The existence of such a first integral generally implies that the system is persistent and has stable steady states. We illustrate our results by examples.

*Keywords:* Darboux polynomial; dynamical system; mass-action; non-linear conservation law; persistence; Lotka-Volterra

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