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Stability and periodic oscillations in the Moon–Rand systems

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ABSTRACT

The Moon–Rand systems, developed to model control of flexible space structures, are systems of differential equations on $\mathbb{R}^3$ with polynomial or rational right hand sides that have an isolated singularity at the origin at which the linear part has one negative and one pair of purely imaginary eigenvalues for all choices of the parameters. We give a complete stability analysis of the flow restricted to a neighborhood of the origin in any center manifold of the Moon–Rand systems, solve the center problem on the center manifold, and find sharp bounds on the number of limit cycles that can be made to bifurcate from the singularity when it is a focus. We generalize the Moon–Rand systems in a natural way, solve the center problem in several cases, and provide sufficient conditions for the existence of a center, which we conjecture to be necessary.

1. Introduction

In [1] (see also Exercise 5 of Section 5.5 of [2]) Moon and Rand introduced the following system of differential equations, which we shall call the *Moon–Rand system*, in the context of modeling control of flexible structures:

$$\begin{aligned}\dot{u} &= v\\ \dot{v} &= -u - uw\\ \dot{w} &= -\lambda w + f(u,v)\end{aligned} \tag{1.1}$$

where

$$f(u,v) = c_{20}u^2 + c_{11}uv + c_{02}v^2 \quad \text{or} \quad f(u,v) = \frac{c_{11}uv}{1+\eta u^2}.$$

Here λ, η, c_{20}, c_{11} and c_{02} are real numbers, $\lambda > 0$, $\eta > 0$. They showed that in the former (polynomial feedback) case the origin is asymptotically stable for the flow restricted to the center manifold if

$$2c_{20} - 2c_{02} - \lambda c_{11} < 0.$$

This condition was found by approximating the local center manifold W^c of (1.1), transforming the system restricted to W^c to a normal form by means of an unspecified near-identity transformation, and going over to polar coordinates.

In this paper we give a complete stability analysis of the flow restricted to a neighborhood of the origin in any center manifold. We allow arbitrary values of η and negative values of λ, requiring only that λ be nonzero so that the singularity

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