ON THE CENTER PROBLEM FOR DEGENERATE SINGULAR POINTS OF PLANAR VECTOR FIELDS

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The center problem for degenerate singular points of planar systems (the *degenerate-center problem*) is a poorly-understood problem in the qualitative theory of ordinary differential equations. It may be broken down into two problems: the *monodromy* problem, to decide if the singular point is of focus-center type, and the *stability* problem, to decide whether it is a focus or a center.

We present an outline on the status of the center problem for degenerate singular points, explaining the main techniques and obstructions arising in the study of the problem. We also present some new results. Our new results are the characterization of a family of vector fields having a degenerate monodromic singular point at the origin, and the computation of the generalized first focal value for this family V_1. This gives the solution of the stability problem in the monodromic case, except when $V_1 = 1$. Our approach relies on the use of the blow-up technique and the study of the blow-up geometry of singular points. The knowledge of the blow-up geometry is used to generate a bifurcation of a limit cycle.

Keywords: Planar vector fields; degenerate singular points; blow-up; focus-center problem; limit cycle.