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Periodic travelling waves in nonlinear reaction–diffusion equations via multiple Hopf bifurcation ☆

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Abstract

We keep track of periodic wave trains for some classes of one dimensional nonlinear reaction–diffusion partial differential equations. These periodic wave trains can be seen as limit cycles of some planar differential systems. We use standard techniques within the multiple Hopf bifurcation framework.

1. Introduction

In this paper we study the existence of *periodic travelling wave solutions* for some non-degenerate one dimensional reaction–diffusion equations of the form

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[D(u)\frac{\partial u}{\partial x}\right] + f(u, u_x), \tag{1}$$

where the diffusion function is such that $D(u) > 0$, for all $u \geqslant 0$. This type of equation arises in many mathematical models for the study of biological and chemical systems, as well as in heat transfer (see [12,17,24,30,34] and references therein, for example).

A *travelling wave* of equation (1) is a solution of the form $u(x,t) = \phi(x - ct)$. A *periodic wave train* (PWT, from now on) is a travelling wave which corresponds to a periodic function ϕ. The existence of these type of solutions can be determined by studying the main features of a system of ordinary differential equations called *travelling wave system*. This system is obtained as follows: Set $u(x,t) = \phi(\xi)$ (where $\xi = x - ct$), by direct substitution of this expression in Eq. (1), we obtain

$$D(\phi)\ddot{\phi} + c\dot{\phi} + D'(\phi)(\dot{\phi})^2 + f(\phi, \dot{\phi}) = 0,$$

where $\{\dot{}\} := \mathrm{d}/\mathrm{d}\xi$. Taking $v := \dot{\phi}$, we have

$$\begin{cases} \dot{\phi} = v, \\ D(\phi)\dot{v} = -cv - f(\phi, v) - D'(\phi)v^2. \end{cases}$$

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