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Note on the Markus–Yamabe conjecture for gradient dynamical systems

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Abstract

Let $v:\mathbb{R}^n \to \mathbb{R}^n$ be a C^1 vector field which has a singular point O and its linearization is asymptotically stable at every point of $\mathbb{R}^n$. We say that the vector field v satisfies the Markus–Yamabe conjecture if the critical point O is a global attractor of the dynamical system $\dot{x} = v(x)$. In this note we prove that if v is a gradient vector field, i.e. $v = \nabla f$ ($f \in C^2$), then the basin of attraction of the critical point O is the whole $\mathbb{R}^n$, thus implying the Markus–Yamabe conjecture for this class of vector fields. An analogous result for discrete dynamical systems of the form $x_{m+1} = \nabla f(x_m)$ is proved.

Keywords: Global attractor; Markus–Yamabe conjecture; Gradient dynamical system

1. Introduction

This paper is related to the problem of providing sufficient conditions in order that a critical point O (the origin of coordinates in what follows) of a C^1 vector field $v:\mathbb{R}^n \to \mathbb{R}^n$ ($v(O) = 0$) be a global attractor (i.e. the ω-limit of any solution of the equation $\dot{x} = v(x)$, $x \in \mathbb{R}^n$, is the

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