

The criticality of centers of potential systems at the outer boundary [☆]

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Abstract

The number of critical periodic orbits that bifurcate from the outer boundary of a potential center is studied. We call this number the criticality at the outer boundary. Our main results provide sufficient conditions in order to ensure that this number is exactly 0 and 1. We apply them to study the bifurcation diagram of the period function of $X = -y\partial_x + ((x+1)^p - (x+1)^q)\partial_y$ with $q < p$. This family was previously studied for $q = 1$ by Y. Miyamoto and K. Yagasaki.

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1. Introduction and setting of the problem

In this paper we study planar differential systems

$$\begin{cases} \dot{x} = f(x, y), \\ \dot{y} = g(x, y), \end{cases}$$

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