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Analytic Tools to Bound the Criticality at the Outer Boundary of the Period Annulus

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Abstract In this paper we consider planar potential differential systems and we study the bifurcation of critical periodic orbits from the outer boundary of the period annulus of a center. In the literature the usual approach to tackle this problem is to obtain a uniform asymptotic expansion of the period function near the outer boundary. The novelty in the present paper is that we directly embed the derivative of the period function into a collection of functions that form a Chebyshev system near the outer boundary. We obtain in this way explicit sufficient conditions in order that at most $n \geqslant 0$ critical periodic orbits bifurcate from the outer boundary. These theoretical results are then applied to study the bifurcation diagram of the period function of the family $\ddot{x} = x^p - x^q$, $p, q \in \mathbb{R}$ with $p > q$.

Keywords Center · Period function · Critical periodic orbit · Bifurcation · Criticality · Chebyshev system

Mathematics Subject Classification 34C07 · 34C23 · 34C25

1 Introduction and setting of the problem

This paper is concerned with the period function of centers of planar differential systems. A singular point p of an analytic differential system

$$\begin{cases} \dot{x} = f(x, y), \\ \dot{y} = g(x, y), \end{cases}$$

is a *center* if it has a punctured neighbourhood that consists entirely of periodic orbits surrounding p. The largest punctured neighbourhood with this property is called the *period*

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