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ISOCHRONICITY OF A CLASS OF PIECEWISE CONTINUOUS OSCILLATORS

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Abstract. Motivated by a classical pendulum clock model suggested by Andrade in 1920, we study the equation $\ddot{x}+g(x)\, sgn\, \dot{x}+x=0$ and prove that for a nonlinear analytic g the origin is never an isochronous focus or an isochronous center.

1. Introduction

The measurement of time (chronometry) has been an enormous incentive for the development of physics and mathematics. An important advance in this field was the discovery of the fundamental properties of the pendulum by Galileo in 1583. Later, many other scientists (especially Huygens and Hooke) took advantage of the natural oscillations of different kinds of pendula and springs in their experiments and designs.

From the mathematical point of view, systems with the property that all solutions are periodic with the same period are called *isochronous*. Isochronous centers, and more generally the study of the so-called period function for potential systems, have received the attention of many researchers [4, 5, 6, 9, 11, 12]. However, from the practical point of view, every classical mechanical device in clockmaking is subjected to a loss of energy due to friction. So, it is natural to look for a notion of *isochronous dissipative oscillator*. In a classical paper [3], Jules Andrade proposed the equation

$$\ddot{x}+\epsilon\,|x|\, sgn\, \dot{x}+x=0 \tag{1}$$

as a first such example with practical applications. Looking at the phase plane, the origin is a global attractor and every non-trivial solution completes a whole turn in a fixed time. The dissipation term $\epsilon\,|x|\, sgn\, \dot{x}$ models the so-called *dry friction* (also

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