

Two inverse problems for analytic potential systems

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Abstract

In this paper, we solve a basic problem about the existence of an analytic potential with a prescribed period function. As an application, it is shown how to extend to the whole phase plane an arbitrary potential defined on a semiplane in order to get isochronicity.

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1. Introduction

In general, an inverse problem could be described as a task where the effect is known, but the cause is unknown. In the framework of dynamical systems, one wonders about the existence of a system which exhibits a concrete dynamic response. This paper is devoted to the analysis of two basic inverse problems in the context of analytic potential systems.

Consider the system

$$\begin{cases} \dot{x} = -y, \\ \dot{y} = V'(x), \end{cases} \quad (1)$$

where V is an analytic function defined in a neighborhood of the origin. We always assume that the system has a non-degenerate center at 0, that is $V(0) = V'(0) = 0$ and $V''(0) = k > 0$. We

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