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Area-Preserving Normalizations for Centers of Planar Hamiltonian Systems

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It is well known that a nondegenerate center of an analytic Hamiltonian planar system can be brought to normal form by means of an analytic canonical change of coordinates. This normal form, that we denote by CNF, does not depend on the coordinate transformation. In this paper we give an elementary proof of these facts and we show some interesting applications of the machinery that we develop in order to prove them. For instance, we describe the space of coordinate transformations that bring a Hamiltonian nondegenerate center to its CNF, and we prove that they are all canonical when the center is non-isochronous. We also show that two Hamiltonian systems with a nondegenerate center are canonically conjugated if and only if both centers have the same period function.

1. INTRODUCTION

The Poincaré Normal Form Theorem (see [9, 13]) asserts that a critical point of an analytic planar differential system is a nondegenerate center if and only if there exists an analytic change of coordinates that brings the initial system to the *normal form*

$$\begin{cases} \dot{x} = -yf(x^2+y^2), \\ \dot{y} = xf(x^2+y^2), \end{cases}$$

where f is an analytic function with $f(0) \neq 0$. We call *normalization* to this coordinate transformation. In case that the differential system is Hamiltonian then this normalization can be chosen *canonical*. By a canonical mapping we mean a mapping such that the determinant of its jacobian is equal to one at any point (i.e., it is area-preserving).

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