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THE BIFURCATION SET OF THE PERIOD FUNCTION OF THE DEHOMOGENIZED LOUD'S CENTERS IS BOUNDED

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Abstract. This paper is concerned with the behaviour of the period function of the quadratic reversible centers. In this context the interesting stratum is the family of the so-called Loud's dehomogenized systems, namely

$$\begin{cases} \dot{x} = -y + xy, \\ \dot{y} = x + Dx^2 + Fy^2. \end{cases}$$

In this paper we show that the bifurcation set of the period function of these centers is contained in the rectangle $K = (-7, 2) \times (0, 4)$. More concretely, we prove that if $(D, F) \notin K$, then the period function of the center is monotonically increasing.

1. Introduction and statement of the result

This paper is devoted to studying the period function of the quadratic centers, and it constitutes the continuation of the results obtained in [14]. There are four families of quadratic centers: Hamiltonian, reversible, codimension four and generalized Lotka-Volterra systems. Chicone has conjectured [2] that the reversible centers have at most two critical periods and that the centers of the three other families have a monotonic period function. The behaviour of the period function of the quadratic centers has been studied extensively, and there is much analytic evidence that the conjecture is true (see [4, 7, 9, 11, 13, 15, 16] and the references therein). It is clear therefore that in this setting the most interesting family of centers is the reversible one. It is well known that any reversible quadratic center can be brought to *Loud's normal form*

$$\begin{cases} \dot{x} = -y + Bxy, \\ \dot{y} = x + Dx^2 + Fy^2, \end{cases}$$

by means of an affine transformation and a constant rescaling of time. It is proved in [5] that if $B = 0$, then the period function of the center is monotonically increasing. The remaining cases, namely $B \neq 0$, can be brought with a rescaling to $B = 1$, i.e.,

$$\begin{cases} \dot{x} = -y + xy, \\ \dot{y} = x + Dx^2 + Fy^2. \end{cases} \tag{1}$$

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