

# Criteria to bound the number of critical periods

F. Mañosas<sup>a,1</sup>, J. Villadelprat<sup>b,\*,2</sup>

<sup>a</sup> *Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona, Spain*

<sup>b</sup> *Departament d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, Tarragona, Spain*

Received 9 May 2008; revised 27 June 2008

Available online 24 July 2008

---

## Abstract

In the present paper we study the period function of centers of potential systems. We obtain criteria to bound the number of critical periods. In case that the system is polynomial, our result enables to tackle the problem from a purely algebraic point of view, since it allows to bound the number of critical periods by counting the zeros of a polynomial. To illustrate its applicability some new and old results are proved.

© 2008 Elsevier Inc. All rights reserved.

MSC: 34C07; 34C08; 34C23

Keywords: Center; Period function; Critical period; Chebyshev system

---

## 1. Introduction and statement of the main result

In this paper we study the period function of centers and we are interested in the case in which it is not monotone, i.e., the center has critical periods. To our knowledge, the key point in almost all the results appearing in the literature dealing with a family of centers with critical periods is that the period function satisfies some kind of Picard–Fuchs differential equation. Let us quote for instance the works of Yulin Zhao for two different families of quadratic centers [24,25] or the papers of Chow and Sanders [4] and Gavrilov [10] on the family of cubic potential centers.

---

\* Corresponding author.

E-mail address: [jordi.villadelprat@urv.cat](mailto:jordi.villadelprat@urv.cat) (J. Villadelprat).

<sup>1</sup> The author is partially supported by the MEC/FEDER grants MTM2005-02139 and MTM2005-06098 and the CIRIT grant 2005SGR-00550.

<sup>2</sup> The author is partially supported by the MEC/FEDER grant MTM2005-06098-C02-01 and the CIRIT grant 2005SGR-00550.