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Bounding the number of zeros of certain Abelian integrals ☆

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ABSTRACT

In this paper we prove a criterion that provides an easy sufficient condition in order for any nontrivial linear combination of n Abelian integrals to have at most $n+k-1$ zeros counted with multiplicities. This condition involves the functions in the integrand of the Abelian integrals and it can be checked, in many cases, in a purely algebraic way.

1. Introduction and statement of the result

In a previous paper with M. Grau [12] we provided a sufficient condition in order for a collection of Abelian integrals $I_0(h), I_1(h), \ldots, I_{n-1}(h)$ to form an extended complete Chebyshev system (for short, ECT-system). This is a very good property that implies in particular that the number of real zeros of any nontrivial linear combination

$$\alpha_0 I_0(h) + \alpha_1 I_1(h) + \cdots + \alpha_{n-1} I_{n-1}(h) \tag{1}$$

counted with multiplicities is at most $n-1$. However there are situations (see for instance [9,10,16, 18]) in which the number of zeros of (1) is greater than $n-1$. Then one talks about being a *Chebyshev*

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