EJECTION–COLLISION ORBITS AND INVARIANT PUNCTURED TORI IN A RESTRICTED FOUR-BODY PROBLEM

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Abstract. We study the motion of an infinitesimal mass point under the gravitational action of three mass points of masses μ, $1 - 2\mu$ and μ moving under Newton's gravitational law in circular periodic orbits around their center of masses. The three point masses form at any time a collinear central configuration. The body of mass $1 - 2\mu$ is located at the center of mass. The paper has two main goals. First, to prove the existence of four transversal ejection–collision orbits, and second to show the existence of an uncountable number of invariant punctured tori. Both results are for a given large value of the Jacobi constant and for an arbitrary value of the mass parameter $0 < \mu \leqslant 1/2$.

Key words: restricted problem, invariant tori, ejection–collision orbits

1. Introduction and Statement of the Main Results

We study the motion of a mass point of negligible mass under the Newtonian gravitational attraction of three mass points of positive mass (called the *primaries*) moving in circular periodic orbits around their center of mass fixed at the origin of the coordinate system. At any instant of time the primaries form a collinear equilibrium configuration of the three-body problem. Two of these primaries have equal masses and are located symmetrically with respect to the third primary which is in rest at the center of mass.

Let m_0, m_1 and m_2 be the masses of the three primaries. We choose the unity of mass in such a way that $m_1 = \mu$, $m_0 = 1-2\mu$ and $m_2 = \mu$, where $\mu \in (0, 1/2)$. The mass m_0 is at rest at the origin of the coordinate system, and the two primaries (of equal masses) are moving in circular orbits around the center of mass of the system. Units of length and time are chosen in such a way that the distances between the primaries m_0 and m_1, and m_0 and m_2, and mean motion of m_1 and m_2 are equal to 1.

For studying the position of the infinitesimal mass m_3 in the plane of motion of the primaries, we use either the *sidereal system* of coordinates (X, Y) (i.e. an inertial system with origin at the mass point m_0), or the *synodical system* of coordinates (x, y) (i.e. a rotational system of coordinates with respect to the sidereal one in which the primaries are in rest). In the synodical system the three point masses m_2, m_0 and m_1 are fixed at $(-1, 0)$, $(0, 0)$ and $(1, 0)$, respectively. In this paper the *restricted*

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